A Study of Interference Fields in a Ducting Environment

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In a cooperative program, the Federal Communications Commission and the Institute for Telecommunication Sciences have begun a study of the interference fields that may arise in a ducting environment. This report gives some preliminary thoughts on how enhanced signal levels might be modeled and describes a measurement program that operated in Southern California. Comparisons are made between some of the first-order statistics of the data and two of the possible models and show that there still remains a large gap between our modeling abilities and reality.

Key words: long-term variability; radio ducts; radio propagation; short-term variability; UHF; VHF

1. INTRODUCTION

In some regions of the world, ducting phenomena are fairly common and lead to interference at surprisingly long distances. To allow better spectrum management, we need to be able to foretell on what propagation paths such long-range interference is likely to happen. This information is essential in managing the broadcast services and the mobile services when one must choose frequency assignments, base station and transmitter locations, and permissible radiated powers and antenna heights.

As a start toward satisfying this need, the Institute for Telecommunication Sciences (ITS) has entered into a cooperative program with the Federal Communications Commission (FCC) to acquire data related to these phenomena, to assemble the resulting statistics, and to try to determine a widely applicable model. A part of these efforts involves a measurement program that originally observed received signal levels on long paths in southern California. The paths extend from Los Angeles to San Diego and are in a region well known for the persistent occurrence of super-refractive layers.

This report gives some preliminary ideas concerning the ultimate model, describes some of the first-order statistics of the data obtained from the measurement program, and provides a comparison of these statistics with candidate models.

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2. POSSIBLE APPROACHES TO MODELING

In attempting to describe the effect of the atmosphere on radio waves, one very straightforward approach is to measure the refractivity structure and then to trace rays or to perform a modal analysis for that structure. Such an approach is actually used today as a real-time technique--by the Navy, for example, to provide a better interpretation of radar returns (see Shkarofsky and Nickerson, 1982), and by NASA to improve the accuracy of range finders. The method is expensive and, as it turns out, still of questionable accuracy. Its biggest drawback, however, is that it provides only a snapshot of the conditions and cannot be directly used for planning purposes.

If we are to have a less expensive, more generally applicable method, we must devise a model for the statistics of received signal levels. Actually, we need two models: one to describe the atmosphere and another to describe the effects of the atmosphere on radio waves. Since the first of these will provide part of the input data to the second, it needs to model only those characteristics of the atmosphere that the second model requires. On the other hand, the second model should not require more data concerning the atmosphere than can be conveniently modeled from meteorological considerations. There is a tight interplay between the two.

There have been many attempts to provide suitable models of the atmosphere. The most recent (and probably most promising) has been that of Dougherty and Dutton (1981). In what follows here, we shall concentrate on the second part of the modeling--the relationship between the atmosphere and received signal levels.

Because the atmosphere changes at the whim of the notoriously hard-to-predict weather, our only choice is to assume that the received signal level on any particular path is to be considered a random process $w(t)$. For future convenience, we measure this in decibels relative to any desired level (1 mW, for example, or 1 μV/m--even the units used are not important for our purposes). And we use the lower-case letter to indicate it is a random variable. Statistics of the process will be indicated by upper-case or Greek letters.

It is customary to separate the signal level process into two superimposed processes. One represents the small-scale or short-term variability, while the other represents the large-scale or long-term variability. The principal reason for this separation is that the two normally arise from different physical causes. The short-term variability is usually due to multipath fading--when there are two or more components of the radio field that arrive at the receiving terminal by way of separate paths and so add together vectorially according to their relative phases. The observed variations with time come about because minute variations in the atmosphere will change the electrical lengths of the several paths and so change the relative phases. Long-term fading, on the other hand, is caused by wide-scale changes in the atmosphere that affect the number and the magnitudes of the multipath components. Since one expects gross changes in the atmosphere during the course of the day, it is common to use the period of one hour to distinguish the two processes. Thus, one also speaks of the within-the-hour and the hour-to-hour variations.

In formal terms we may write

$$w(t) = w_0(t) + r(t) \tag{1}$$

where $w_0(t)$ is a "smoothed" version of the received signal level (usually "hourly medians") and the residual $r(t)$ represents the small-scale variations. As indicated by the lower-case letters, both components are to be treated as random processes; w_0 is measured in the same units as is w, and r is measured in simple decibels. Since $w_0(t)$ will equal the local median value, r will always have a median value of 0 dB.

The form of (1) implies a multiplicative relation for the corresponding amplitudes or powers. This is by design. If the short-term variations are caused by multipath, then they may also be called "frequency-selective fading." If at any instant we were to examine the received signal level over a wide band of frequencies, we would find a very similar kind of variability in which $w_0(t)$ provides an overall, general level. From the point of view of the resulting frequency transfer function, it would be quite proper to multiply the general level by a normalized random function of frequency.

2.1. Short-term Variability

In the case of short-term variability there are some simple considerations that already allow us to reach fairly concrete results. These considerations rest on the assumption that this variability is caused, as stated above, by multipath fading in which the phases of received components vary because of minute variations in the refractivity of the atmosphere. Statistics of the short-term variability then depend only on the number of components and on their relative magnitudes.

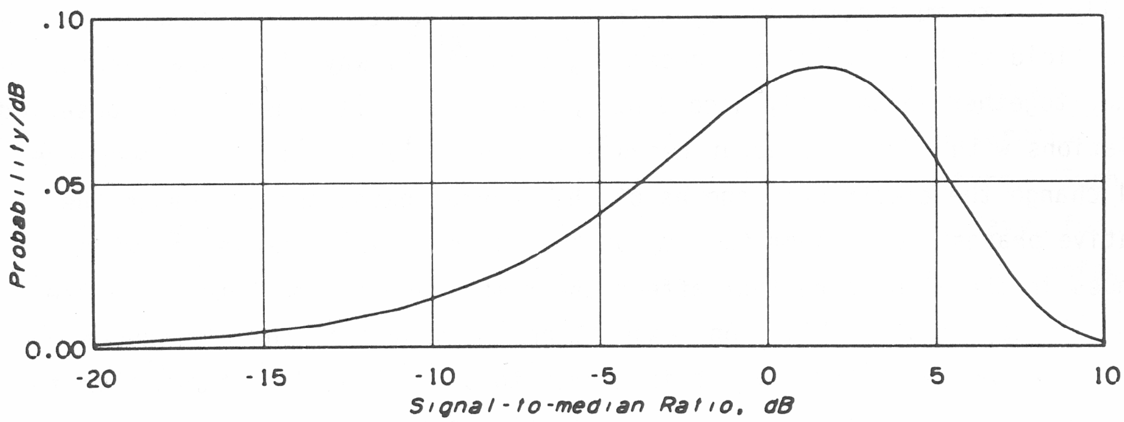

Figure 1. The density function of the Rayleigh decibel distribution.

First, let us suppose there are many components, none of which predominate. Then we have the case of Rayleigh fading (see, e.g., Rayleigh, 1894). The received signal is the vector sum of several voltages having nearly the same amplitude but entirely random phases. The first-order statistics (those that depend only on a single instant of time and do not measure how rapidly the signal varies) satisfy the Rayleigh Law. For the quantity r, the (complementary) cumulative distribution is given by the simple formula

$$q = \Pr[r > R] = 2^{-10^{R/10}} \tag{2}$$

We might call this the Rayleigh decibel distribution since the term "Rayleigh distribution" is usually restricted to the distribution of the underlying voltage amplitudes. Solving (2) for R we obtain the quantile

$$R(q) = 10 \log \frac{\ln 1/q}{\ln 2} \tag{3}$$

so that R(q) is the value that r exceeds for the fraction q of the time. Note that this distribution has no parameters, a fact that makes it easy to use since no measurements to determine parameters are necessary. It has a mean of 0.92 dB, a standard deviation of 5.57 dB, and an interdecile range $\Delta r = R(0.1) - R(0.9) = 13.40$ dB. In Figure 1 we have plotted its density function--the negative derivative of (2). The resulting bell-shaped curve is skewed toward the negative values.

Of course, it is more common in the literature to introduce a nonzero median as a simple parameter of the Rayleigh distribution. In our notation this is done by simply adding that median to the right-hand side of (3); here, this addition is taken care of already in (1) by the local median term w_0.

When the multipath components do not satisfy the simple requirements for Rayleigh fading, the next condition one can assume is that there are still several components present but that one of them is much stronger than the others. This condition leads to first-order statistics that satisfy what is known as the Nakagami-Rice distribution (and also the Ricean or the "constant-plus-a-Rayleigh" distribution; see, e.g., Norton et al., 1955). Aside from the median (which here we shall want to assume vanishes), it has one parameter that can be represented as C, the "constant-to-scattered" ratio measured in decibels. This is the ratio of the power in the single strong component (the "constant" component) to the average power in the sum of the remaining components (the "scattered" component). It is interesting to note that, except for a slight change in meanings of the variables and parameters, the Rayleigh distribution is equivalent to the chi-squared distribution with two degrees of freedom. Similarly, the Nakagami-Rice distribution is equivalent to the "noncentral" chi-squared distribution with two degrees of freedom. Both of these distributions are described in, e.g., Abramowitz and Stegun (1964), and computer programs to calculate values may be found in some of the available libraries of mathematical or statistical functions. In any case, however, statistics of the Nakagami-Rice distribution are hard to compute and they depend on the parameter C in a nontrivial way.

In studies of short-term variability, it is customary to plot observed cumulative distributions on "Rayleigh paper." This is graph paper in which the ordinate represents the received signal level scaled linearly in decibels, and the abscissa represents probability (or fraction of time) scaled nonlinearly according to the negative of the right-hand side of (3). If a distribution satisfies the Rayleigh Law, then it will appear on Rayleigh paper as a straight line with slope -1.

On the other hand, if we used Rayleigh paper to plot the Nakagami-Rice distribution, we obtain results such as those portrayed in Figure 2 where the several curves correspond to several values of the parameter C. The most

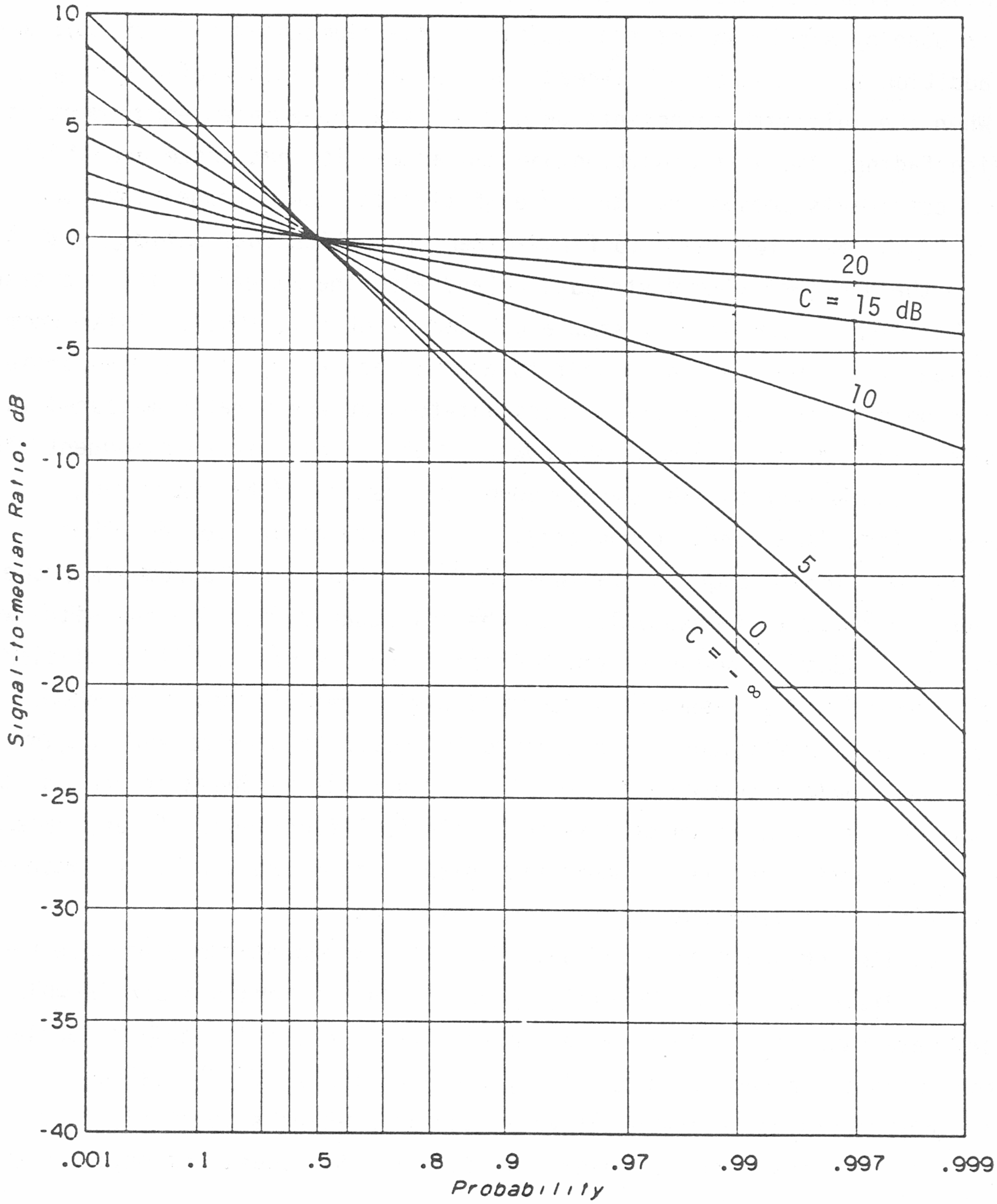

Figure 2. Nakagami-Rice distributions drawn on Rayleigh paper. The parameter C is the constant-to-scattered ratio in decibels.

striking thing about these curves is that they all look like straight lines; so much so that replacing them by straight lines appears to be a reasonable approximation.

Now as it happens there is a distribution used in the theory of reliability known as the Weibull distribution, which may be most simply defined as one that plots out as a straight line of arbitrary slope on Rayleigh paper (Weibull, 1951). It seems very appropriate, then, to use this distribution as an approximation to the Nakagami-Rice distribution, the point being that many of its characteristics are easy to compute. If the slope equals $-\alpha$, then the quantiles are easily determined from (3) as

$$R(q) = \alpha \, 10 \, \log \frac{\ln 1/q}{\ln 2} \tag{4}$$

As with the Rayleigh distribution, we should probably call this the Weibull decibel distribution, since the usual definition is in terms of the amplitude. The mean, standard deviation, and interdecile range are all equal to the Rayleigh values multiplied by the slope α. Since the relation here is so simple and since the slope of a cumulative distribution is one measure of the "concentration," one may as well replace α as the parameter of the Weibull distribution by either the standard deviation or the interdecile range, both of which may have more immediate meanings. In what follows, we shall normally prefer the interdecile range Δr as the most convenient of these measures.

To remain within the range of the Nakagami-Rice distribution, the value of α should be restricted to lie between 0 and 1. At the extreme $\alpha = 0$, we have a simple jump distribution in which $r(t) = 0$ with probability 1. This corresponds to the case where there is only the isolated single component, the others having vanished. At the other extreme, $\alpha = 1$, we have the Rayleigh distribution again; here, the isolated component has vanished. For intermediate values there will be a nontrivial relation between the parameter C of the Nakagami-Rice distribution and the slope. In Figure 3 we have plotted the interdecile range of the Nakagami-Rice distribution versus C. This plot is, however, of only incidental interest since from a phenomenological point of view it is immaterial which of these related parameters we use and the interdecile range is certainly the easier to measure and to employ in further computations.

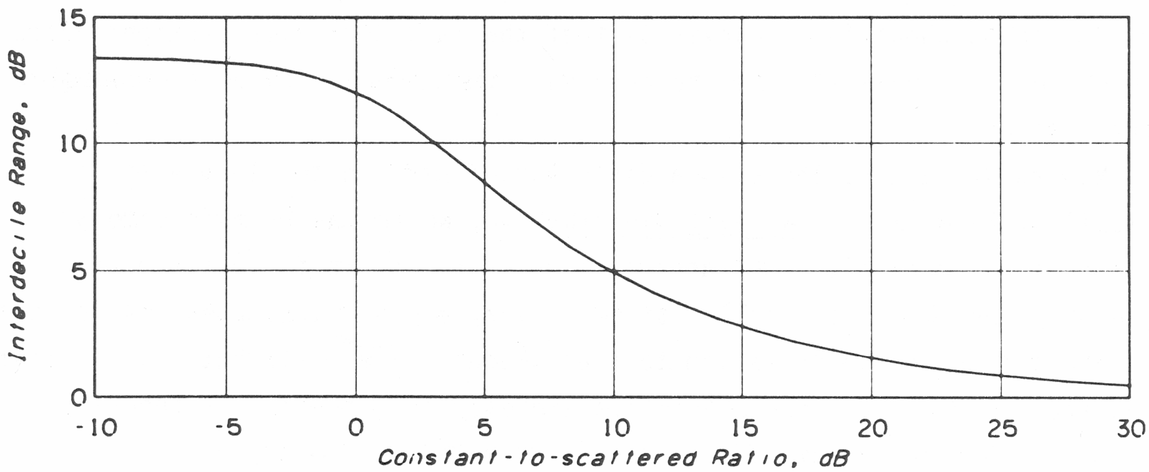

Figure 3. The interdecile range of the Nakagami-Rice distribution.

There is one other model of the first-order statistics of short-term variability that is sometimes mentioned in the literature (see, for example, Dougherty, 1968). Called the two-ray model, it assumes there are just two components in the multipath field with relative phases that are uniformly distributed over the whole circle. The resulting quantiles (in decibels) are given by

$$R(q) = 10 \log\left(1 + \frac{2\beta}{1+\beta^2} \cos \pi q\right) \tag{5}$$

where β is the (voltage) ratio between the amplitudes of the two components. In Figure 4 we have plotted a few of these distributions on Rayleigh paper using the parameter $B = 20 \log \beta$. In addition to flattening out in the tails, these distributions all show a strong downward curvature at central values. We might note that when B vanishes (so that the two components have equal magnitudes) the interdecile range is 16.01 dB. This is therefore one example of a distribution with a steeper average slope than that of the Rayleigh distribution. The difference, however, is small and when B exceeds only 2.5 dB, it vanishes.

Experience has shown that in the great majority of cases the Nakagami-Rice distribution (and especially the limiting case of the Rayleigh distribution) does indeed portray fairly accurately the observed first-order statistics of short-term fading. Until other distributions, such as that of

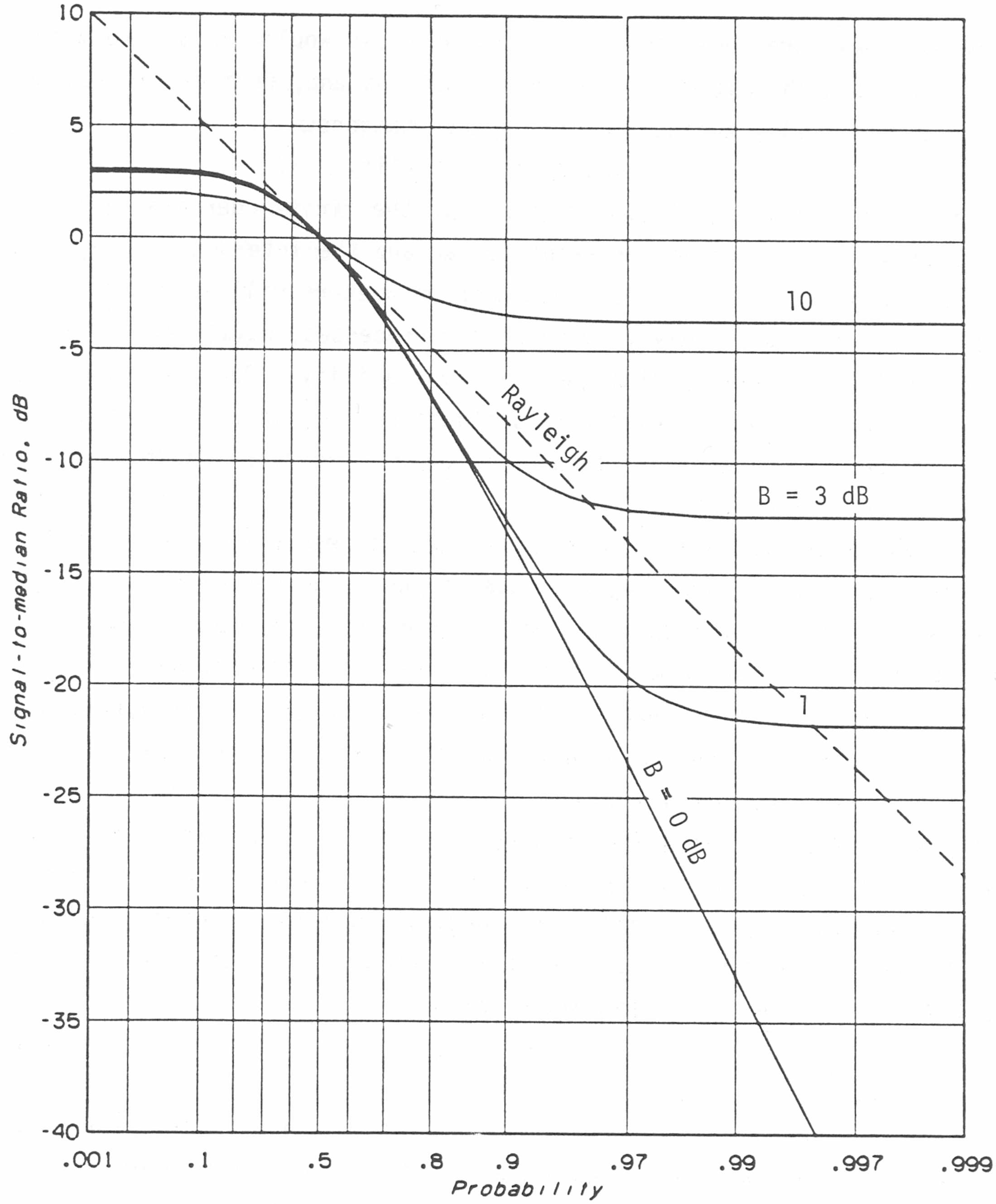

Figure 4. Distributions of the two-ray model drawn on Rayleigh paper. The parameter B is the ratio in decibels of the two amplitudes.

the two-ray model, are shown to be of importance, we would propose to model these statistics with this one set of distributions and, further, to replace them by the more easily managed Weibull approximations.

2.2 Long-Term Variability

We have seen, then, that within each hour the first-order statistics can be characterized by two parameters--the median and the interdecile range Δr. From hour to hour we would suppose that both parameters will vary and so we are led to consider two slowly varying random processes, $w_0(t)$ and $\Delta r(t)$, whose statistics are those of the long-term variability. Our first concern will be for the first-order statistics of these processes; but note that we would expect the two to be statistically dependent and also that they probably have diurnal and seasonal trends.

To study these statistics, we seem obliged to resort to an empirical study of measured data. Our present state of knowledge does not permit us to do much more, although certainly any modeling of the phenomena should take account of whatever we do know. There have been very few previous measurements of the within-the-hour interdecile range and an attempt to model its statistics must wait on the acquisition of a body of data. We turn here to a discussion of the hourly median.

Of past attempts at modeling long-term variability, probably the most widely used method is that suggested by Rice et al. (1967). It forms a part of the ITS Irregular Terrain Model (see Longley and Rice, 1968; and Hufford et al., 1982) and of a Comité Consultatif International des Radiocommunications (CCIR, 1978a) approach. In this method one writes

$$w_0(t) = W_{ref} + v(t) \tag{6}$$

where W_{ref} is a "reference" level and $v(t)$ is a "deviation." The reference level is meant to be the fixed signal one would obtain with a "normal" atmosphere--i.e., a hydrodynamically unstable (hence turbulent) atmosphere where the refractivity follows a fixed exponential decrease with increasing altitude. The deviation then forms a random process, and there is nothing new here. The important step, however, is to assume that the first-order statistics of the deviations can be described in terms of only two parameters--the "climatological type" and an "effective distance." The climatological type can be one of some eight discrete types ranging from "equatorial" to "polar." The effective distance is a function of actual